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- 1928 - Elected Corresponding Member of Paris Academy of Sciences. Published articles on absolutely monotonic functions (Acta Mathematica, 1928) and submitted several reports to the Boulogne Congress of Mathematicians dealing with regular monotonic functions.
- 1928 - Given Directorship of newly created Ukrainian Institute of Mathematics, whose founding he advocated.
- 1929 - Gave series of lectures on orthogonal polynomials at Poincare Institute in Paris and Polytechnical School in Zurich.
- 1929 - Elected Active Member (Academician) of Academy of Sciences USSR.
- 1930 - Appointed Chairman of Committee for the Organization of the First All-Union Conference of Mathematicians.
- 1933 to 1940 - Resided in Leningrad and worked actively in Academy of Sciences USSR; conducted courses at Leningrad State University and Leningrad Industrial School.
- 1935 - Conducted studies at the Chair of the Theory of Probability, Scientific Research Institute of Leningrad State University.
- 1945 - Awarded Order of Lenin (USSR) and degree of "Doctor Honoris Causa" (Sorbonne, France).
- 1948 - Until April of this year, was chief of Division of Theory of Probability and Mathematical Statistics, Department of Physical-mathematical Sciences, Academy of Sciences USSR; replaced in April by Academician A. N. Kolmogorov /no reason given for this change/.
- 1948 - In December, awarded a 10,000-ruble prize from Presidium, Academy of Sciences USSR, for his work, "Approximate Functions of Finite Degree."
- 1950 - In March, celebrated his 70th birthday.

Bernshteyn follows the traditions of the great Russian mathematicians Chebyshev, Markov, and Lyapunov, as well as the French school of mathematics represented by Picard, Hadamard, and Vallee Poussin, as reflected in his work. At the basis of his creative genius lies an unshakable belief that the mathematical method is the key to natural sciences. "Today Mathematics and physics are in agreement; and the field of mathematical applications is limited only by our own limits of knowledge" -- such were the introductory words to Bernshteyn's first work (1903), in which he solved one of the most famous mathematical problems to attract great mathematicians, originally suggested by D. Hilbert at the 1900 Paris Mathematics Congress. In this work, Bernshteyn proved the analyticity of all sufficiently "smooth" solutions of second-order partial differential equations of the elliptic type. Bernshteyn has always opposed idle play of the intellect. All his works are closely connected with problems of natural science in its widest sense.

Another characteristic of Bernshteyn's creative works is his preference for concrete, difficult problems over generalizations. Overcoming great difficulties lying in the way of his solving a problem, Bernshteyn

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often outstrips his contemporaries by seizing those fields whose systematic development belongs to the future. At first glance it always seems that Bernshteyn solves one difficult particular problem. Actually, it turns out that those methods by which the solution is obtained are employed for solving a very wide class of problems.

Bernshteyn is credited with a large amount of remarkable works. Their number has continued to grow quickly up to the present. Without pretending to any complete discussion of them, we shall now take up the more important of their trends.

As we have said, his first works were devoted to the theory of differential equations. After the above-mentioned work on the analyticity of solutions of elliptic-type equations, followed a great cycle of related works devoted to proving the existence of the solution of Dirichlet's problem for wide classes of nonlinear elliptic equations. The results of these works in many respects remain unsurpassed even now. Also related here is the following theorem, which is closely connected to the generalization of the Liouville theorem on boundary harmonic functions defined in the entire plane: if a function is defined in the entire real plane, then the surface $z = f(x, y)$ cannot for any h be contained between the planes $z = \pm h$ if its Gaussian curvature is everywhere not positive and not identically equal to zero.

Later, Bernshteyn's main attention was directed to the theory of approximation of functions by algebraic and trigonometric polynomials, and later still, to problems of approximation of functions given on the entire infinite axis, by means of entire (integral) functions of finite degree. The pioneer in this field was the famous P. L. Chebyshev. After him, these works were carried on by Ye. I. Zolotarev, A. A. Markov, and V. A. Markov. To Bernshteyn belongs credit for numerous results in this field which are of first-rate importance. These works of his served as the basis for the present-day constructive theory of functions of a real variable. Let us note some of them.

It is known that for every continuous function $f(x)$ defined on a finite interval there exists, for every positive integer n , a polynomial $P_n(x)$ of degree n whose deviation from $f(x)$, that is, $\max |f(x) - P_n(x)|$, possesses a minimum in comparison with the deviation from $f(x)$ of all other polynomials of the n th degree. This minimum deviation is designated by $E_n(f)$. Bernshteyn showed a method which enables one to construct with any given degree of accuracy a polynomial of the n th degree which polynomial deviates the least from $f(x)$. He investigated the connection between the rapidity of decrease of $E_n(f)$ as n approaches infinity with the smoothing of the function $f(x)$; he showed a method which enables one to establish the differential properties of functions if knowledge of their optimum approximation is available. He showed that analytic functions of a real variable are characterized by the fact that $E_n(f)$ tends to zero, for such functions, more quickly than C^n , where C is a certain positive number. Weakening of this condition led Bernshteyn to important classes of quasi-analytical functions; that is, to those functions whose values on the entire interval are completely determined by their value on a part, as small as desired, of this interval. Let us also note still finer evaluations of $E_n(f)$ and the remarkable polynomials shown by Bernshteyn and bearing his name:

$$B_n(x) = \sum_{k=0}^n f(k/n) C_n^k x^k (1-x)^{n-k}.$$

They uniformly approximate, as n approaches infinity, any continuous function $f(x)$ defined on the interval $(0, 1)$.

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Associated with the problems of approximating a function by algebraic polynomials is the theory of approximation of functions by means of trigonometric polynomials, as well as the theory of trigonometric series and the theory of mechanical quadrature, in which Bernshteyn again obtained many remarkable results. We note, for example, the following widely known theorem of his: if a function satisfies the Lipschitz condition for index (exponent) less than one half, then it can be expanded into an absolutely convergent Fourier series.

Very significant results were obtained by Bernshteyn in a study of the extremum properties of polynomials. His inequalities for the derivatives of trigonometric polynomials, which now bear his name, were generalized in various ways. Particularly important generalizations are credited to Bernshteyn.

Recently Bernshteyn has obtained a number of remarkable results in the theory of approximating a function, which are defined on the infinite axis, by means of entire (integral) functions of finite degree.

The third group of problems taken up by Bernshteyn is the theory of probability. Initial results in this science, just as in the theory of approximations of functions by polynomials, belong to the Russian scientists P. L. Chebyshev, A. A. Markov, and A. M. Lyapunov. Bernshteyn's works on the theory of probability brilliantly carry on the researches of these scientists on the limiting theorems for sums of chance quantities.

He proved the basic limiting theorem for sums of independent chance quantities for very wide assumptions and established wide conditions for which the limiting theorem holds for sums of dependent chance quantities. He was the first also to prove the limiting theorem for the two-dimensional case. To these fundamental results Bernshteyn has also added his elegant and less general solutions of a great number of more particular problems in the theory of probability and mathematical statistics.

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